Kinematic Analysis of a Planar Tensegrity Mechanism

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ABSTRACT

This paper presents the equilibrium analysis of a planar tensegrity mechanism. The device consists of a base and top platform that are connected by one connector leg (whose length can be controlled via a prismatic joint) and two spring elements whose linear spring constants and free lengths are known. The paper presents two cases, one where the spring free lengths are both zero, and the other where the spring free lengths are nonzero. The purpose of the paper is to show the enormous increase in complexity that results from nonzero free lengths.

Keywords

planar mechanisms, tensegrity

1. INTRODUCTION

The word tensegrity is a combination of the words tension and integrity (Edmondson, 1987 and Fuller, 1975). Tensegrity structures are spatial structures formed by a combination of rigid elements in compression (struts) and connecting elements that are in tension (ties). No pair of struts touch and the end of each strut is connected to three non-coplanar ties (Yin et al, 2002). The entire configuration stands by itself and maintains its form solely because of the internal arrangement of the struts and ties (Tobie, 1976).

The development of tensegrity structures is relatively new and the works related have only existed for approximately twenty five years. Kenner, 1976, established the relation between the rotation of the top and bottom ties. Tobie, 1976, presented procedures for the generation of tensile structures by physical and graphical means. Yin, 2002, obtained Kenner's results using energy considerations and found the equilibrium position for unloaded tensegrity prisms. Stern, 1999, developed generic design equations to find the lengths of the struts and elastic ties needed to create a desired geometry for a symmetric case. Knight, 2000, addressed the problem of stability of tensegrity structures for the design of deployable antennae.

2. PROBLEM STATEMENT

The mechanism to be analyzed here is shown in Figure 1. The top platform (indicated by points 4, 5, and 6) is connected to the base platform (indicated by points 1, 2, and 3) by two spring elements whose lengths are L_1 and L_2 and by a variable length connector whose length is referred to as L_3. Although this does not match the exact definition of tensegrity, the device is

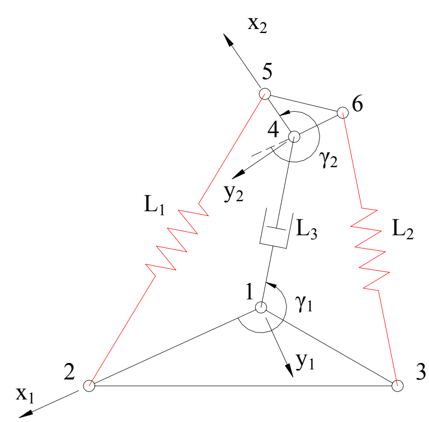

Figure 1. Compliant Mechanism

prestressed in the same manner as a tensegrity mechanism. The exact problem statement is as follows:

given:

- L_{12} distance between points 1 and 2
- p_{3x}, p_{3y} coordinates of point 3 in coord. system 1
- L_{45} distance between points 4 and 5
- p_{6x}, p_{6y} coordinates of point 6 in coord. system 2
- L_3 distance between points 1 and 4
- k_1, L_{01} spring constant and free length of spring 1
- k_2, L_{02} spring constant and free length of spring 2

find:

- all static equilibrium configurations

It is apparent that since the length L_3 is given, the device has two degrees of freedom. Thus there are two descriptive parameters that must be selected in order to define the system. For this analysis, the descriptive parameters are chosen as the angles γ_1 and γ_2 which are shown in Figure 1. Other parameters were investigated, but none yielded a less complicated solution than is presented here.

3. SOLUTION APPROACH

Two possible solution approaches were considered, i.e. (1) satisfy force and moment conditions for equilibrium and (2) obtain configurations of minimum potential energy. Each approach was found to realize the same set of constraint equations. As such, obtaining the condition for force and moment balance is presented here.

The first step of the analysis is to determine the coordinates of the six points in terms of the base coordinate system as expressed in terms of the descriptive parameters γ_1 and γ_2. The coordinates of the three points in the base may be written as

$$\mathbf{P}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{P}_2 = \begin{bmatrix} L_{12} \\ 0 \end{bmatrix}, \mathbf{P}_3 = \begin{bmatrix} p_{3x} \\ p_{3y} \end{bmatrix}. \tag{1}$$

The coordinates of the three points in the top platform may be written as

$$\mathbf{P}_4 = \begin{bmatrix} L_3\, c_1 \\ L_3\, s_1 \end{bmatrix}, \mathbf{P}_5 = \begin{bmatrix} L_3\, c_1 + L_{45}\, c_2 \\ L_3\, s_1 + L_{45}\, s_2 \end{bmatrix}, \quad \mathbf{P}_6 = \begin{bmatrix} L_3\, c_1 + p_{6x}\, c_2 - p_{6y}\, s_2 \\ L_3\, s_1 + p_{6x}\, s_2 + p_{6y}\, c_2 \end{bmatrix} \tag{2}$$

where s_i and c_i, i=1,2, represent the sine and cosine of the angle γ_i.

A free body diagram of the top platform indicates that the sum of the forces along the three connector lines must equal zero at equilibrium. The unitized Plücker coordinates of a connector line can be obtained as

$$\mathbf{\$}_i = \frac{1}{d_i}\begin{bmatrix} x_t - x_b \\ y_t - y_b \\ \left[(x_b\mathbf{i} + y_b\mathbf{j}) \times \left((x_t - x_b)\mathbf{i} + (y_t - y_b)\mathbf{j}\right)\right]\cdot\mathbf{k} \end{bmatrix} \tag{3}$$

where (x_t, y_t) and (x_b, y_b) are respectively the coordinates of the points on the top and bottom platforms that are on the line and d_i is the distance between the points that is calculated as

$$d_i = \sqrt{\left(x_t - x_b\right)^2 + \left(y_t - y_b\right)^2}\ . \tag{4}$$

Thus the unitized Plücker coordinates of the three connector lines may be written as

$$\mathbf{\$}_1 = \frac{1}{d_1}\begin{bmatrix} L_3\, c_1 + L_{45}\, c_2 - L_{12} \\ L_3\, s_1 + L_{45}\, s_2 \\ L_{12}(L_3\, s_1 + L_{45}\, s_2) \end{bmatrix} \tag{5}$$

$$\mathbf{\$}_2 = \frac{1}{d_2}\begin{bmatrix} L_3\, c_1 + p_{6x}\, c_2 - p_{6y}\, s_2 - p_{3x} \\ L_3\, s_1 + p_{6x}\, s_2 + p_{6y}\, c_2 - p_{3y} \\ p_{3x}(L_3\, s_1 + p_{6x}\, s_2 + p_{6y}\, c_2) - p_{3y}(L_3\, c_1 + p_{6x}\, c_2 - p_{6y}\, s_2) \end{bmatrix} \tag{6}$$

$$\mathbf{\$}_3 = \begin{bmatrix} c_1 \\ s_1 \\ 0 \end{bmatrix} \tag{7}$$

where

$$d_1 = \sqrt{2L_3L_{45}(c_1c_2 + s_1s_2) - 2L_{12}(L_3c_1 + L_{45}c_2) + L_{12}^2 + L_3^2 + L_{45}^2} \tag{8}$$

$$d_2 = \text{sqrt}\ \{2L_3(p_{6x}s_2 + p_{6y}c_2 - p_{3y})s_1 + 2(p_{6y}p_{3x} - p_{6x}p_{3y} - L_3p_{6y}c_1)s_2 + 2L_3(p_{6x}c_2 - p_{3x})c_1 + L_3^2 + p_{3x}^2 + p_{3y}^2 + p_{6x}^2 + p_{6y}^2\}\ . \tag{9}$$

The force in each of the springs can be written as

$$f_1 = k_1\,(d_1 - L_{01}) \tag{10}$$

$$f_2 = k_2\,(d_2 - L_{02})\ . \tag{11}$$

The summation of the three forces that are acting on the top platform may be written as

$$f_1\mathbf{\$}_1 + f_2\mathbf{\$}_2 + f_3\mathbf{\$}_3 = \mathbf{0}\ . \tag{12}$$

It is interesting to note that this equation implies that a necessary condition for static equilibrium is that the three line coordinates are linearly dependent.

The three line coordinates may now be expressed as

$$\mathbf{\$}_i = \begin{bmatrix} l_i \\ m_i \\ n_i \end{bmatrix}, i = 1..3 \tag{13}$$

where the terms l_i, m_i, and n_i have been defined in (5), (6), and (7).

Equation (12) may be rearranged as

$$f_3\begin{bmatrix} l_3 \\ m_3 \\ n_3 \end{bmatrix} = -\begin{bmatrix} f_1 l_1 + f_2 l_2 \\ f_1 m_1 + f_2 m_2 \\ f_1 n_1 + f_2 n_2 \end{bmatrix}. \tag{14}$$

In order for a solution to exist, it is necessary that the three scalar equations represented by (14) be satisfied. Since $n_3 = 0$, it may be written that

$$f_1 l_1 + f_2 l_2 = 0\ . \tag{15}$$

Eliminating the unknown f_3, from the two scalar equations obtained from the first two rows of (14) gives

$$l_3\,(f_1 m_1 + f_2 m_2) - m_3\,(f_1 m_1 + f_2 m_2) = 0\ . \tag{16}$$

Equations (15) and (16) represent the conditions that must be satisfied for the mechanism to be in static equilibrium. All the terms in these equations have been defined in terms of the descriptive parameters γ_1 and γ_2.

4. ZERO FREE LENGTH CASE

4.1 Analysis

For this simplified case it is assumed that the free lengths of the two springs, i.e. L_{01} and L_{02}, are both equal to zero. The forces in the two springs as defined in (10) and (11) now reduce to

$$f_1 = k_1\, d_1 \tag{17}$$

$$f_2 = k_2\, d_2\ . \tag{18}$$

Substituting these expressions as well as the line coordinate terms defined in (5) and (6) into (15) and (16) give

$$L_3\,(k_1L_{12} + k_2p_{3x})\, s_1 + [k_1L_{12}L_{45} + k_2(p_{3x}p_{6x} + p_{3y}p_{6y})]\, s_2 - k_2L_3p_{3y}\, c_1 + k_2\,(p_{3x}p_{6y} - p_{3y}p_{6x})\, c_2 = 0 \tag{19}$$

$$(k_1L_{45} + k_2p_{6x})\,(c_1s_2 - s_1c_2) + k_2p_{6y}\,(c_1c_2 + s_1s_2) + (k_2p_{3x} + k_1L_{12})\, s_1 - k_2p_{3y}\, c_1 = 0\ . \tag{20}$$

Note that when the free lengths of the springs are zero that the terms d_1 and d_2 vanish.

The solution for the values of the angles γ_1 and γ_2 that simultaneously satisfy (19) and (20) proceeds by defining their tan-half angles as

$$x_i = \tan\frac{\gamma_i}{2} \tag{21}$$

and then introducing the trigonometric identities

$$s_i = \frac{2x_i}{1+x_i^2},\ c_i = \frac{1-x_i^2}{1+x_i^2}. \tag{22}$$

Substituting (22) into (19) and (20) and rearranging yields

$$(A_1x_2^2+A_2x_2+A_3)x_1^2 + (A_4x_2^2+A_5x_2+A_6)x_1$$
$$+ (A_7x_2^2+A_8x_2+A_9) = 0\ , \tag{23}$$
$$(B_1x_2^2+B_2x_2+B_3)x_1^2 + (B_4x_2^2+B_5x_2+B_6)x_1$$
$$+ (B_7x_2^2+B_8x_2+B_9) = 0 \tag{24}$$

where the coefficients A_1 through B_9 are expressed in terms of given quantities as

$$\begin{aligned}
A_1 &= k_2(p_{6x}p_{3y} - p_{6y}p_{3x} + L_3p_{3y})\ ,\\
A_2 &= 2k_1L_{12}L_{45} + 2k_2(p_{3x}p_{6x} + p_{3y}p_{6y})\ ,\\
A_3 &= k_2(p_{6y}p_{3x} - p_{6x}p_{3y} + L_3p_{3y})\ ,\\
A_4 &= 2(k_1L_{12}L_3 + k_2L_3p_{3x})\ ,\\
A_5 &= 0\ ,\\
A_6 &= A_4\ ,\\
A_7 &= k_2(p_{6x}p_{3y} - p_{6y}p_{3x} - L_3p_{3y})\ ,\\
A_8 &= A_2\ ,\\
A_9 &= k_2(-p_{6x}p_{3y} + p_{6y}p_{3x} - L_3p_{3y})\ ,
\end{aligned} \tag{25}$$

$$\begin{aligned}
B_1 &= k_2(p_{6y} + p_{3y})\ ,\\
B_2 &= -2(k_2p_{6x} - 2k_1L_{45})\ ,\\
B_3 &= k_2(p_{3y} - p_{6y})\ ,\\
B_4 &= 2k_1(L_{45} + L_{12}) + 2k_2(p_{3x} + p_{6x})\ ,\\
B_5 &= 4k_2p_{6y}\ ,\\
B_6 &= 2k_1(L_{12} - L_{45}) + 2k_2(p_{3x} - p_{6x})\ ,\\
B_7 &= k_2(-p_{3y} - p_{6y})\ ,\\
B_8 &= 2k_1L_{45} + 2k_2p_{6x}\ ,\\
B_9 &= k_2(p_{6y} - p_{3y})\ .
\end{aligned} \tag{26}$$

Crane and Duffy (1998) present two methods for solving a pair of equations of the form of (23) and (24). Using Bezout's method, the two equations may be written as

$$P_1\, x_1^2 + Q_1\, x_1 + R_1 = 0\ , \tag{27}$$
$$P_2\, x_1^2 + Q_2\, x_1 + R_2 = 0 \tag{28}$$

where

$$P_1 = A_1x_1^2+A_2x_2+A_3\ , \tag{29}$$
$$Q_1 = A_4x_2^2+A_5x_2+A_6\ , \tag{30}$$
$$R_1 = A_7x_2^2+A_8x_2+A_9\ . \tag{31}$$
$$P_2 = B_1x_2^2+B_2x_2+B_3\ , \tag{32}$$
$$Q_2 = B_4x_2^2+B_5x_2+B_6\ , \tag{33}$$
$$R_2 = B_7x_2^2+B_8x_2+B_9\ . \tag{34}$$

The condition that the quadratics (27) and (28) have a common root for x_1 is that

$$\begin{vmatrix} P_1 & Q_1 \\ P_2 & Q_2 \end{vmatrix}\begin{vmatrix} Q_1 & R_1 \\ Q_2 & R_2 \end{vmatrix} - \begin{vmatrix} P_1 & R_1 \\ P_2 & R_2 \end{vmatrix}^2 = 0\,. \tag{35}$$

Since the terms P_1 through R_2 are quadratic with respect to x_2, expansion of (35) yields an eighth degree polynomial in the variable x_2. The coefficients of this polynomial have been obtained symbolically, but are not listed here due to their length. It was found that this eighth degree polynomial could be divided symbolically by the term $(1+x_2^2)$ with no remainder resulting in a sixth order polynomial in the variable x_2.

Values for x_1 that correspond to each of the eight solutions for x_2 can be determined from

$$x_1 = -\frac{\begin{vmatrix} Q_1 & R_1 \\ Q_2 & R_2 \end{vmatrix}}{\begin{vmatrix} P_1 & R_1 \\ P_2 & R_2 \end{vmatrix}}\,. \tag{36}$$

Unique corresponding values for γ_1 and γ_2 were calculated for each value of x_1 and x_2 from (21) as

$$\gamma_i = 2\arctan(x_i),\ i=1,2\ . \tag{37}$$

4.2 Numerical Example

The following values were selected for a numerical example:

- $L_{12} = 9.220$ m,
- $p_{3x} = -3.254$ m, $p_{3y} = 3.796$ m,
- $L_{45} = 1.367$ m,
- $p_{6x} = -0.305$ m, $p_{6y} = -0.882$ m
- $L_3 = 6$ m
- $k_1 = 2$ N/m, $L_{01} = 0$,
- $k_2 = 3.5$ N/m, $L_{02} = 0$.

The coefficients listed in (25) and (26) were evaluated numerically and the coefficients of the sixth order polynomial in the variable x_2 were obtained by expanding (35) and dividing throughout by $(1+x_2^2)$. The six solutions for γ_2 and the corresponding solutions for γ_1 are listed in Table 1.

Table 1. Six Solutions for Zero Free Length Case

Solution #	γ_1, radians	γ_2, radians
1	-2.2905	-2.6125
2	-1.8510	0.3100
3	1.0441	2.2880
4	1.1136	-0.8197
5	-1.7543 + 0.0641 i	1.1886 – 0.7453
6	–1.7543 – 0.0641 i	1.1886 + 0.7453

The four real solutions are shown in Figure 2. The two complex solutions were shown to satisfy equations (19) and (20).

5. NONZERO FREE LENGTH CASE

5.1 Analysis

The problem statement and solution approach here is the same as presented in Sections 2 and 3. Now, however, the free lengths of the springs, i.e. L_{01} and L_{02}, are nonzero. Substituting (10) and (11) as well as the line coordinate terms defined in (5) and (6) into (15) and (16) and rearranging now gives

$$\mathcal{A}_1 d_1 d_2 + \mathcal{A}_2\, d_1 + \mathcal{A}_3\, d_2 = 0\ , \tag{38}$$

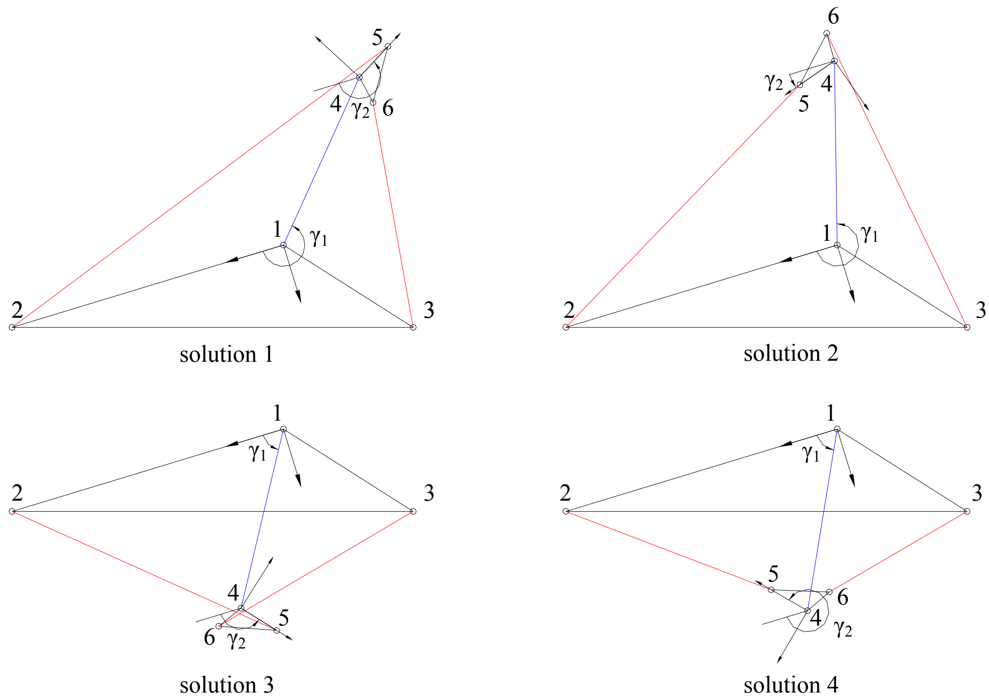

Figure 2. Four Real Solutions

$$B_1 d_1 d_2 + B_2 d_1 + B_3 d_2 = 0 \tag{39}$$

where the terms d_1 and d_2 are functions of the angles γ_1 and γ_2 as defined in (8) and (9) and A_1 through B_3 are also functions of the angles γ_1 and γ_2 and are defined as

$$\begin{aligned}
A_1 &= L_3(k_1L_{12}+k_2p_{3x})s_1 - k_2p_{3y}L_3c_1 \\
&\quad + (k_1L_{12}L_{45}+k_2p_{3x}p_{6x}+k_2p_{3y}p_{6y})s_2 + k_2(p_{3x}p_{6y}-p_{3y}p_{6x})c_2 , \\
A_2 &= -k_2L_{02}p_{3x}L_3s_1 + k_2L_{02}p_{3y}L_3c_1 \\
&\quad - k_2L_{02}(p_{3x}p_{6x}+p_{3y}p_{6y})s_2 + k_2L_{02}(p_{3y}p_{6x}-p_{3x}p_{6y})c_2 , \\
A_3 &= -k_1L_{12}L_{01}(L_3s_1+L_{45}s_2) ,
\end{aligned} \tag{40}$$

$$\begin{aligned}
B_1 &= k_2p_{6y}(c_1c_2+s_1s_2) - (k_1L_{45}+k_2p6x)(s_1c_2-c_1s_2) \\
&\quad + (k_1L_{12}+k_2p_{3x})s_1 - k_2p_{3y}c_1 , \\
B_2 &= -k_2L_{02}p_{6y}(c_1c_2+s_1s_2) + k_2L_{02}p_{6x}(s_1c_2-c_1s_2) \\
&\quad -k_2L_{02}p_{3x}s_1 + k_2L_{02}p_{3y}c_1, \\
B_3 &= k_1L_{01}L_{45}(s_1c_2-c_1s_2) - k_1L_{01}L_{12}s_1 .
\end{aligned} \tag{41}$$

The difficulty in obtaining solutions to equations (38) and (39) is that the terms d_1 and d_2 (which were not present in the previous zero spring free length case) contain the square root of terms containing the sines and cosines of γ_1 and γ_2. To address this problem, (38) and (39) are rearranged as

$$A_2 d_1 = -(A_1 d_1 + A_3)\, d_2 , \tag{42}$$

$$B_2 d_1 = -(B_1 d_1 + B_3)\, d_2 . \tag{43}$$

Squaring these equations yields

$$A_2^2 d_1^2 - d_2^2\,(A_1^2\, d_1^2 + 2A_1A_3d_1 + A_3^2) = 0 , \tag{44}$$

$$B_2^2 d_1^2 - d_2^2\,(B_1^2\, d_1^2 + 2B_1B_3d_1 + B_3^2) = 0 . \tag{45}$$

These two equations can now be rearranged as

$$A_2^2 d_1^2 - d_2^2\,(A_1^2\, d_1^2 + A_3^2) = 2\, A_1A_3d_1d_2^2 , \tag{46}$$

$$B_2^2 d_1^2 - d_2^2\,(B_1^2\, d_1^2 + B_3^2) = 2\, B_1B_3d_1d_2^2 , \tag{47}$$

Squaring (46) and (47) yields

$$\{A_2^2 d_1^2 - d_2^2\,(A_1^2\, d_1^2 + A_3^2)\}^2 - \{2\, A_1A_3d_1d_2^2\}^2 = 0 , \tag{48}$$

$$\{B_2^2 d_1^2 - d_2^2\,(B_1^2\, d_1^2 + B_3^2)\}^2 - \{2\, B_1B_3d_1d_2^2\}^2 = 0 . \tag{49}$$

All the d_1 and d_2 terms in (48) and (49) have been raised to an even power. This allows for the substitution of (8) and (9) into (48) and (49) without the existence of any square root term.

Since the terms A_1 through B_3 and d_1^2 and d_2^2 are first order in the sines and cosines of γ_1 and γ_2, expansion of (48) and (49) will yield two equations that are of degree 8 in the sines and cosines of γ_1 and γ_2. Substitution of the tan-half angle expressions defined in (22) will result in two equations that are 16th degree in the parameters x_1 and x_2. Utilizing Sylvester's Dialytic method, that is described in Crane and Duffy (1998), to eliminate the parameter x_1 would yield a single polynomial in x_2 of degree 512. This is a significant increase in complexity compared to the degree 6 polynomial that resulted from the zero free length case.

5.2 Solution via the Continuation Method

The continuation method (Garcia and Li, 1980, Morgan, 1983, 1986, 1987, Wampler et al., 1990) is a numerical technique to solve a set of equations in multiple variables. This is as opposed to Sylvester's method which would lead to a symbolic solution of the problem.

A concise description of the continuity method is presented by Tsai, 1999. Suppose one wishes to solve the set of equations F(x) which are defined by

$$F(\mathbf{x}) : \begin{cases} f_1(x_1, x_2, \cdots, x_n) = 0 \\ f_2(x_1, x_2, \cdots, x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n) = 0 \end{cases} . \tag{50}$$

F(**x**) is called the target system.

The continuation method begins by first estimating the total number of possible solution sets (sets of values for x_1 and x_2 for this case) that satisfy the given equations. For example, Bezout's theorem states that a polynomial of total degree n has at most n isolated solutions in the complex Euclidean space. Including solutions at infinity, the Bezout number of a polynomial system is equal to the total degree of the system.

Next, an initial system, G(**x**) =**0**, is obtained, whose solution will be of the same degree as that of F(**x**), but whose solution set is known in closed form. In other words, G(**x**) maintains the same polynomial structure as F(**x**).

Finally, a homotopy function H(**x**, t) is prepared such as

$$H(\mathbf{x}, t) = \gamma\,(1-t)\,G(\mathbf{x}) + t\,F(\mathbf{x}) \tag{51}$$

where γ is a random complex constant. When t=0, the homotopy function equals the initial system, G(**x**). When t=1, the homotopy function equals the target system, F(**x**). Recall that the solutions to G(**x**) are known. As the parameter t is increased in small steps from 0 to 1, the solutions of H(**x**, t) can be tracked (referred to as path tracking) and when t =1, these solutions will be the solutions to the original target system. If the degree of the solution set was overestimated, some of the solutions will track to infinity and these can easily be discarded.

5.3 Numerical Example

The following values were selected for a numerical example:

- L_{12} = 9.220 m, p_{3x} = -3.254 m, p_{3y} = 3.796 m,
- L_{45} = 1.367 m, p_{6x} = -0.305 m, p_{6y} = -0.882 m
- L_3 = 6 m
- k_1 = 2 N/m, L_{01} = 11.5 m,
- k_2 = 3.5 N/m, L_{02} = 9.5393 m .

Numerical coefficients were obtained based on this input data set for the equations (48) and (49). The continuation method was run on this set of two equations in two unknowns to obtain all solution sets for the two variables, x_1 and x_2, for the particular numerical example. The software PHC pack (Verschelde (1999)) was used to implement the continuation method.

The software estimated that there would be 512 total solution sets. A total of 462 solutions were found to converge. The entire set of solutions is not presented here due to the large number. Of these solutions, 12 were real. These solutions are being evaluated to determine if they do represent equilibrium solutions for the mechanism. The complex solutions are being checked to verify that they satisfy equations (48) and (49).

6. CONCLUSIONS

This paper has presented an approach to determine all equilibrium configurations of a planar mechanism comprised of two bodies connected by a variable length connector leg and two spring elements. The approach is simple, i.e. define the problem by two descriptive parameters and then obtain the equations that represent the conditions for static equilibrium. Here a simple force and moment balance approach was used. As shown in the paper, the solution is trivial when the free lengths of the two springs are zero. However, the complexity of the problem makes the solution virtually unmanageable when non zero free lengths are considered.

7. ACKNOWLEDGMENTS

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