Robotic Arm Manipulator Control for SG5-UT

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ABSTRACT
This paper presents a complete forward and inverse kinematics solution for SG5-UT, 5 DOF robotic arm. The solution is intended to be implemented on a microprocessor to control the arm in any environment. The control presented in the paper makes it possible to manipulate the arm to any reachable position. The algorithm derived in this paper has been successfully tested on the arm. This arm is analyzed for the purposes of being mounted on a humanoid robot, called Gnuman.

Keywords
Inverse Kinematics; Manipulator Control; Robotic Arm

1. INTRODUCTION
The forward and inverse kinematics for control of the SG5-UT robotic arm (Figure 1) is developed in this paper. This work can also be applied to any robotic manipulators of similar configuration.

After a thorough review of preliminary research previously compiled in the field of robotics, a complete kinematics solution for this arm was determined. Another aim of this research is to develop the programming to implement the kinematic solutions on the arm.

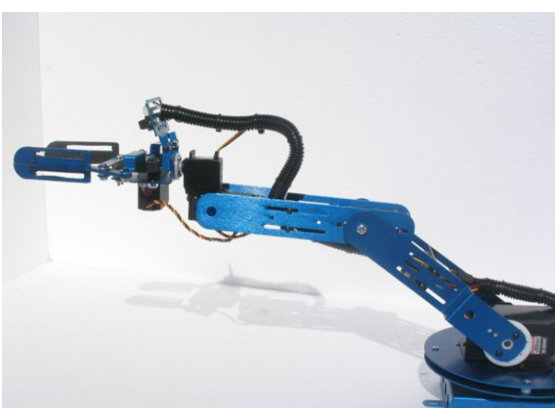

Figure 1: SG5-UT, 5DOF Robotic Manipulator

Utilization of a microprocessor control board in order to manipulate the arm is one of the key aspects of this research. Software packages like MATLAB were used extensively in order to design the simulations and to perform the necessary kinematics analysis.

The SG5-UT is made by Crust Crawler (www.crustcrawler.com). It has five (5) degrees of freedom. In Robotics, degrees of freedom (DOF) are the set of independent rotations or displacements that specify completely the position and orientation of the body or the system. This is a fundamental concept relating to systems of moving bodies in robotic arms' mechanics.

The SG5-UT is governed by two servos for the bicep joint. One servo provides the degree of freedom allocated to the elbow joint. The remaining two servos are utilized in the gripper. Figure 2 depicts the configuration of SG5-UT while fully stretched and initialized to zero degrees at each joint. This also defines the initial frame.

2. KINEMATICS

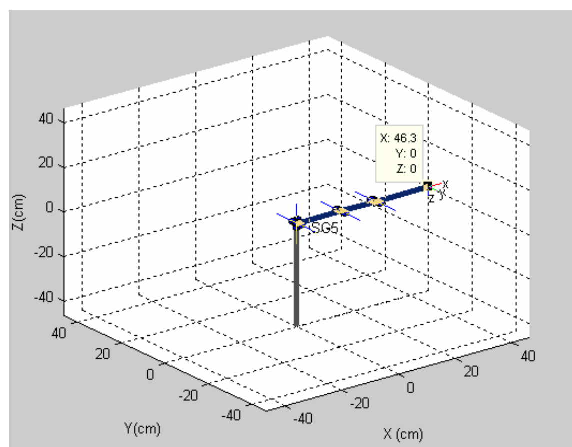

Figure 2: Robot in fully stretched position

Inverse kinematics modeling has historically been one of the foremost tribulations in robotics research. When the inverse kinematics is not performed, a popular method for controlling robotic arms is still based on look-up tables that are usually designed in a manual manner. Alternative methods include neural networks and optimal search. The approach used in this paper is based on analytical inverse kinematics for the SG5-UT. There are no previous controllers for this arm which apply a closed loop inverse kinematics solution. Inverse kinematics is unique due to the reason that the position parameters and orientation parameters can be defined separately, hence providing the liberty to chose position precision at the expense of orientation and vice versa. Inverse kinematics solutions provide a more robust controller for trajectory generation and movement of the robot as compared to other methods. The iterative method to solve for the inverse kinematics is popular for the manipulators which do not have a closed form solution available. The iterative process is more taxing on resources and is less accurate than the process involving the closed form solutions.

2.1 Forward Kinematics

In order to represent the inverse kinematics of the robotic manipulator, the Denavit-Hartenberg (D-H) convention is used. The D-H parameters can be used to model robot joints and links for any serial link manipulator, regardless of its complexity[1]. The illustration in Figure 3 defines the end effector position which is the center of the arm gripper.

The D-H parameters can be found by assigning a local frame reference at every joint as shown in Figure 3. Physical measurements on the robot were made to extract the D-H parameters in Table 1. The a_i parameter is the distance along x_i from the origin to the intersection of the x_i and z_{i-1} axes. Similarly, d_i is the distance along z_{i-1} from the origin to the intersection of x_i and z_{i-1} axes. It is variable if the join (i) is prismatic. α_i is the angle between z_{i-1} and z_i measured about x_i and θ_i is the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint (i) is revolute.

Table 1: D-H Parameters

Link	α_i	a_i(cm)	d_i	θ_i
1	$\pi/2$	0	0	θ_1
2	0	a_2=15.5	0	θ_2
3	0	a_3=12.3	0	θ_3
4	$\pi/2$	a_4=18.5	0	θ_4

The transformation matrix can be formed by using D-H parameters for forward and inverse kinematics of the manipulator[5].

The following matrix represents the rotation about the z_{i-1} axis by θ_i , then about the x_i axis by α_i..

$$ {}^{i-1}R_i = R_z(\theta_i)R_x(\alpha_i) = \begin{bmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i & c\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i \\ 0 & s\alpha_i & c\alpha_i \end{bmatrix} \quad (1) $$

$$ = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i \\ 0 & s\alpha_i & c\alpha_i \\ 0 & 0 & 0 \end{bmatrix} \quad (2) $$

Below, the equation represents the translation of z_i axis a distance of d_i followed by the translation along x_{i-1} axis a distance of a_i.

$$ {}^{i-1}\mathbf{d}_{i-1,i} = d_i\,{}^{i-1}\mathbf{z}_{i-1} + a_i\,{}^{i-1}\mathbf{x}_i = d_i\,{}^{i-1}\mathbf{z}_{i-1} + a_i\,{}^{i-1}R_i\,{}^{i}\mathbf{x}_i $$

$$ = d_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a_i \begin{bmatrix} c\theta_i \\ s\theta_i \\ 0 \end{bmatrix} = \begin{bmatrix} a_i c\theta_i \\ a_i s\theta_i \\ d_i \end{bmatrix} \quad (3) $$

The overall translation and rotation transformation from one frame to another is given below. The SG5-UT robot has no d_i parameters since all of the joints are revolute and all displacements are orthogonal to joint rotations.

$$ {}^{i-1}T_i = \begin{bmatrix} {}^{i-1}R_i & {}^{i-1}d_{i-1,i} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4) $$

The total transformation between the base of the robot and the end effector is given by:

$$ {}^0T_4 = {}^0T_1 . {}^1T_2 . {}^2T_3 . {}^3T_4 \quad (5) $$

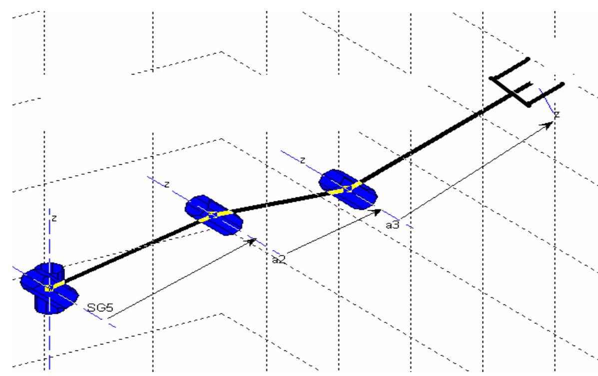

Figure 3: Frames for D-H Parameters

$$ {}^0T_4 = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6) $$

The forward kinematics solution of the SG5-UT is given by the product of the 4 transformation matrices, where p_x, p_y, and p_y are the global coordinates indicating the spatial position of the center of end effector in fully open position and n_x, n_y , n_z, o_x , o_y , o_z , a_x, a_y , and a_z represent the global orientation parameters that follow the D-H convention.

By equating the product of the four matrices with the total transformation, ${}^0T_4$, a set of 12 equations that define our forward kinematics are found.

$$ n_x = C_1(C_{23}C_4 - S_{23}S_4) \quad (7) $$

$$ n_y = S_1(C_{23}C_4 - S_{23}S_4) \quad (8) $$

$$ n_z = S_{23}C_4 - C_{23}S_4 \quad (9) $$

$$o_x = S_1 \tag{10}$$

$$o_y = -C_1 \tag{11}$$

$$o_z = 0 \tag{12}$$

$$a_x = C_1(C_{23}S_4 - S_{23}C_4) \tag{13}$$

$$a_y = S_1(C_{23}S_4 - S_{23}C_4) \tag{14}$$

$$a_z = S_{23}S_4 - C_{23}C_4 \tag{15}$$

$$p_x = C_1(a_4(C_{23}C_4 - S_{23}S_4) + a_3C_{23} + C_2a_2) \tag{16}$$

$$p_y = S_1(a_4(C_{23}C_4 - S_{23}S_4) + a_3C_{23} + C_2a_2) \tag{17}$$

$$p_z = a_4(S_{23}C_4 - C_{23}S_4) + a_3S_{23} + S_2a_2) \tag{18}$$

In the above equations $S_i = \sin(\theta_i)$, $S_{ij} = \sin(\theta_i + \theta_j)$, $C_i = \cos(\theta_i)$, and $C_{ij} = \cos(\theta_i + \theta_j)$.

It is important to note that o_z is zero because of the fact that the end effector lacks a degree of freedom in order to achieve the desired orientation. Under strict robotics terminology, SG5-UT is not really a 5 DOF arm. It is a 4 DOF manipulator and has a degree of freedom in the gripper, which doesn't play any role in the orientation or the positioning of the arm in the kinematics terms.

2.2 Inverse Kinematics

Inverse kinematics is the process of determining the unknown parameters of a serial linked rigid body object in order to achieve a desired orientation and position. This issue is vital in robotics, where manipulator arms are commanded in terms of joint angles (or displacements). The inverse kinematics solutions involve applying various symbolic manipulation techniques to determine a closed form (when possible) solution for the angles (or displacements) with respect to the orientation and position coordinates.

Dividing 17 and 18, we get

$$\frac{p_y}{p_x} = \tan(\theta_1) \tag{19}$$

$$\theta_1 = atan2(p_y, p_x) \tag{20}$$

This is a simple solution of the base angle.

In order to calculate θ_2 and θ_3, from (10), (11) and (12), we can derive the following set of equations:

$$\frac{p_x - a_4n_x}{C_1} = a_3C_{23} + C_2a_2 \tag{21}$$

$$\frac{p_y - a_4n_y}{S_1} = a_3C_{23} + C_2a_2 \tag{22}$$

$$p_z - a_4n_z = a_3S_{23} + a_3S_2 \tag{23}$$

Setting,

$$r_1 = a_3C_{23} + C_2a_2 \tag{24}$$

$$r_2 = a_3S_{23} + S_2a_2 \tag{25}$$

Squaring the above equations and adding them yield:

$$a_3^2 + 2a_2a_3(C_{23}C_2 + S_{23}S_2) + a_2^2 = r_1^2 + r_2^2 \tag{26}$$

Where

$$\begin{aligned} C_{23}C_2 + S_{23}S_2 &= cos(\theta_2) = -cos(\pi - \theta_2) \\ &= cos(\theta_3 - \pi) = cos(-\theta_3) \end{aligned} \tag{27}$$

Expanding 13 and 14,

$$r_1 = (a_3C_{23} + a_3)C_2 - a_3S_3S_2 \tag{28}$$

$$r_2 = (a_3C_{23} + a_3)S_2 - a_3S_3C_2 \tag{29}$$

The following pair of solutions is achieved.

$$\theta_2 = atan2(r_2, r_1) - acos\left(\frac{r_1^2 + r_2^2 + a_2^2 - a_3^2}{2a_2\sqrt{r_1^2 + r_2^2}}\right) + 2n\pi \tag{30}$$

Where n can be -1,0,1 making θ_2 between $-\pi$ to π.

And corresponding θ_3 becomes:

$$\theta_3 = \left[\pi - acos\left(\frac{-r_1^2 - r_2^2 + a_2^2 + a_3^2}{2a_2a_3}\right)\right] \tag{31}$$

And another set of possible solutions is:

$$\theta_2 = atan2(r_2, r_1) + acos\left(\frac{r_1^2 + r_2^2 + a_2^2 - a_3^2}{2a_2\sqrt{r_1^2 + r_2^2}}\right) + 2n\pi \tag{32}$$

$$\theta_3 = \left[-\pi + acos\left(\frac{-r_1^2 - r_2^2 + a_2^2 + a_3^2}{2a_2a_3}\right)\right] \tag{33}$$

It should be noted that $r \neq 0$, which means the above equations always provide certain solutions for θ_2 and θ_3.

The solution for θ_4 can be found by manipulating (9) and (15) as a function of the previously determined θ_1, θ_2, and θ_3.

We get,

$$C_4 = \frac{S_{23}S_4 - a_z}{C_{23}} \tag{34}$$

$$C_4 = \frac{n_z - C_{23}S_4}{S_{23}} \tag{35}$$

Equating both sides we get,

$$S_4 = n_z C_{23} + a_z S_{23} \tag{36}$$

This implies,

$$\theta_4 = atan2(n_z C_{23} + a_z S_{23}, n_z S_{23} - a_z C_{23}) \tag{37}$$

This concludes the unique set of two solutions to the inverse kinematics of the SG5-UT arm. Our strategy for choosing the correct solution is to calculate all the two sets of possible solutions (joint angles). Thus, two possible corresponding positions and orientations will be generated using forward kinematics. A comparison between the positions found by the forward kinematics and the desired position can be made. Hence, the solution with minimal error can be ultimately chosen.

Theoretically, the equations for calculating joint angles θ_1 to θ_4 are correct. However, in practice there could be problems in atan2 and acos calculations. For instance, due to numerical problems with finite precision calculations, the absolute value of the variable in acos could be slightly greater than 1.The workspace of the robotic arm is depicted in Figure 4. It consists of a spherical surface, which accounts for the outer boundary of the reachable space, and a cylindrical surface, which depicts the inner boundary of reachable space. The points on the surface depict singularities of the arm.

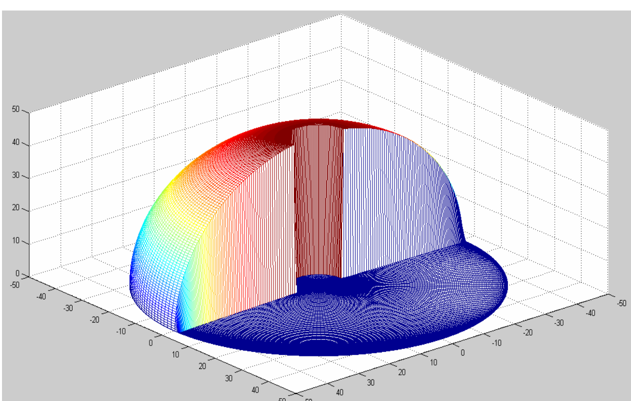

Figure 4: Workspace of the SG5-UT

3. APPLICATIONS

The inverse kinematics solution can be programmed with a microprocessor. This will generate real-time conditions in turn, manipulating the arm. Sensory data should be interpreted in a manner that provides the inverse kinematics function within a microprocessor. All of the orientation and position coordinates should remain consistent with the D-H convention. The microprocessor can interpret the angles into servo parameters that are sent to the servo controller module. Figure 5 depicts a block diagram that incorporates the control process. A sensor, such as a CMU camera, can send the position parameters to the microprocessor, which then computes the inverse kinematics and sends the joint parameters to a servo controller. The servo controller moves the arm at the right speed making a trajectory for the arm movement.

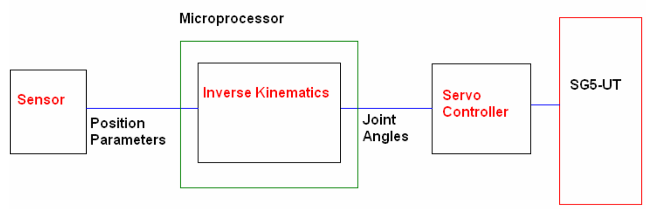

Figure 5: Block Diagram for arm control

The derived algorithm can be used in a low cost 8-bit microprocessor using a lookup table for the sine and cosine. The tangent can be calculated using the sine and cosine terms. The accuracy of the calculated angles is compromised but is sufficient for the servos for the purposes of this robot. A Taylor series approximation can also be used to computer sine and cosine more accurately at the cost of resources, such as microprocessor clock speed. The algorithm based on the inverse kinematics method has been tested and proves sufficient for the arm.

The SG5-UT was selected because of its commercial availability and its ability to satisfy all the project goals. The control of this arm has been developed for the humanoid robot called Gnuman (see Figure 6). Gnuman's platform is being developed and is founded on a tripod which will allow the robot to travel in any direction with no explicit "front" or "back". Each wheel is independent of the others in both steering and drive which allows the robot to move in a hypotrochoid (spin) fashion as it moves. The design also showcases three arms, which makes the Gnuman a dynamic oriented robot.

When the Gnuman is equipped with three (3) robotic arms, it will be further able to complete one of its basic goals, i.e., making a presentation while being able to point at various other robots. The presentation will involve both speech and manipulation of a computer mouse. A set of three (3) SG5-UT arms can fulfill the needs of Gnuman, and as a consequence, make it a more powerful and dynamic robot.

4. CONCLUSION

A complete analytical solution to the inverse kinematics of SG5-UT is derived for the first time in this paper. The derived analytical inverse kinematics model always provides the correct joint angles for manipulation of the arm end-effector to any given reachable position and orientation. Even if the given position/orientation(s) cannot be reached to the exact level, the model is able to give a superior level of approximate solutions. We believe that the solution developed in this document will make the SG5-UT more useful in applications with unpredictable trajectory movements in unknown environments. Without this elucidation, the trajectory movements of the SG5-

UT would have to be completed by manually manipulating the arm to follow the trajectory and recording a sequence of joint angles for the later use in the trajectory following task. The analytical solution is able to automatically provide joint angles for a given trajectory in an efficient, accurate, and effective manner.

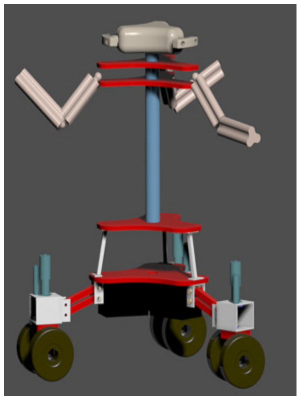

Figure 6 : Design of a humanoid robot, Gnuman

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