

# Using the Wave Variable Method for a Human-Machine Haptic Interface in the Presence of Time Delay

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## Abstract

Even a small amount of time delay in a bilateral teleoperation system will generally degrade the system's performance and cause instability. Consequently, without some form of compensation for time delay, latencies in a teleoperation system would preclude the use of force feedback. Fortunately, there are approaches based on scattering theory and passivity that can compensate for time delay and allow the use of force feedback in teleoperation systems with latencies. In particular, the wave variable method is a passivity-based approach that guarantees stability for any fixed time delay. In this work, the authors take a generalized approach which includes the complete family of scaling matrices. This extended family of scaling matrices is used in an experiment with human subjects and a PHANToM Omni haptic teleoperation system. The experiment will compare the raw data with the users' opinions in order to determine the best set of scaling matrices for the given task.

## Keywords

Teleoperation, haptics, human-machine interfaces, time delay, wave variables

## 1. INTRODUCTION

### 1.1 Background on Teleoperation

Since the introduction of the first modern master/slave manipulator in the late 1940's, teleoperation systems have been used for a number of different tasks, e.g., handling toxic or harmful materials, operating in remote environments such as undersea or space, and performing tasks that require extreme precision, and will continue to play an increasingly important role for such applications in the future [8]. A bilateral teleoperator is a dual robot system in which a remote slave robot tracks the motion of a master robot. In haptic teleoperation, a human operator commands the master robot, and force information is communicated back from the slave to the master.

### 1.2 Problems with Time Delay

One major problem that can be found in bilateral teleoperation systems is caused by time delay. When the master and slave are in close proximity, time delay may not be an issue. However, when the master and slave are located at a far distance from each other, the time delay is no longer negligible. Unfortunately, even a small time delay in a bilateral teleoperation system will generally degrade the system's performance and cause instability [4-7], [19], [24].

### 1.3 Wave Variables

In the late 1980's, Anderson and Spong showed that it is possible to stabilize a force reflecting teleoperation system that has a time delay by exploiting scattering theory [2]. Later, Niemeyer and Slotine presented the wave variable method as a more intuitive, physically motivated formalism based on passivity [18]. Today, the standard control architecture for bilateral teleoperation systems is based on the scattering theory formalism used in [2] and subsequently reformulated using wave variables in [18].

In the wave variable method, wave variables are used in place of more conventional power variables like velocity and force. It was found that when forces and velocities were transformed into wave variables and transmitted at both the master and slave sides, the overall system could remain stable even with time delay. This powerful approach is based on the concept of passivity, an extremely important property that can be effectively used to ensure the overall stability of a connected system of passive subsystems.

In spite of the recent advances in the area of constant time delay, issues regarding stability and performance of systems with variable time delay still remain a challenge that must be addressed if teleoperation is to reach its full potential. Such issues have been the motivation of recent research work on extending the wave variable method. For example, Munir and Book included predictive techniques in the wave variable method to handle the time-varying delays encountered on the Internet [14]-[17]. Other work on controlling teleoperators experiencing variable time delay includes [3], [13], [20], [26].

In addition to including predictive techniques in their wave variable architecture, Munir and Book [14]-[17] also introduced a generalization of the wave variables using a set of scaling matrices to work with multiple degree-of-freedom systems. Although their generalization is nontrivial, it is not fully general. In previous work, we examined this generalization of the wave variable method to multiple degree-of-freedom teleoperators in more detail and extended the choice of the wave parameters to the complete family of such scaling matrices.

### 1.4 Haptic Experiment

Even though several experiments have been done in the area of teleoperation and haptics [10-12], [21], [23], [25], none have used the extended wave variable method as a means of dealing with time delay. In this article we will use the complete family of scaling matrices in an experiment involving human subjects. We developed a bilateral teleoperation system with force feedback

using a PHANToM Omni haptic device as both master and slave. Subjects were asked to complete several tasks using the master PHANToM. Each trial used a different set of scaling matrices as well as one of two different amounts of time delay. During the experiment, the subjects were asked several questions about their experience with the haptic device. The answers to these questions will be used in conjunction with the raw data in order to determine the best sets of scaling matrices for each amount of time delay.

## 2. THE WAVE VARIABLE METHOD

### 2.1 Bilateral Teleoperation

In a basic bilateral setup, a human operator commands a master robot, which sends information to a slave manipulator, which in turn sends force feedback to the master that the operator can feel. As long as no time delay is present, this system performs well, i.e., the slave's behavior tracks the master's behavior. If even a small amount of delay is introduced into the system, the performance will quickly degrade and the system may even become unstable unless some sort of compensation is introduced.

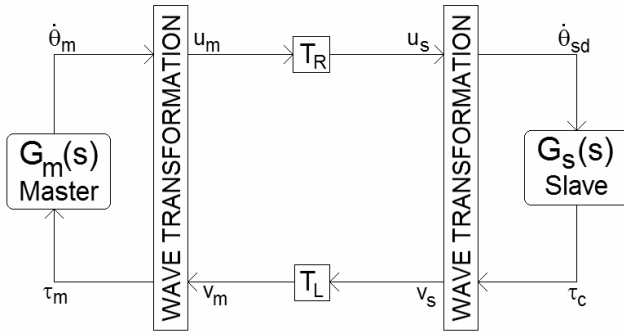


Figure 1. A bilateral teleoperation system with wave transformations.

### 2.2 Single Degree-of-Freedom

The wave variable method is an important approach to mitigating time delay in a bilateral teleoperation system. Figure 1 is an illustration of a haptic bilateral teleoperation system with wave variable transformations present. The parameters  $T_L$  and  $T_R$  represent the time delays in the left and right directions between the master and slave as shown in Figure 1. We will assume that the delay in each direction is the same and so  $T_R = T_L = T$ . The wave transformation relations for the single degree-of-freedom case are given by

$$u_s(t) = u_m(t - T) \quad (1)$$

$$v_m(t) = v_s(t - T).$$

The wave transformations for the left wave junction are given by

$$u_m(t) = \frac{b\dot{\theta}_m(t) + \tau_m(t)}{\sqrt{2b}} \quad (2)$$

$$v_m(t) = \frac{b\dot{\theta}_m(t) - \tau_m(t)}{\sqrt{2b}}$$

and that for the right wave junction are given by

$$u_s(t) = \frac{b\dot{\theta}_{sd}(t) + \tau_c(t)}{\sqrt{2b}} \quad (3)$$

$$v_s(t) = \frac{b\dot{\theta}_{sd}(t) - \tau_c(t)}{\sqrt{2b}}.$$

Although the strictly positive parameter  $b$  can be chosen arbitrarily, it defines a characteristic impedance associated with the wave variables and directly affects the system behavior [18].

### 2.3 Expansion to Multiple Degree-of-Freedom Systems

Equations (2) and (3) are for single degree-of-freedom systems. To implement the wave variable method on a system that has more than one degree of freedom, the equations for the transforms must be generalized. Niemeyer and Slotine [18] suggest making  $b$  a positive definite matrix. Munir and Book [14]-[17] have shown that in going to the higher degree of freedom case, one can use a more general formulation. In particular, they introduced the following form for the wave transformation equations:

$$u_m(t) = A_w \dot{\theta}_m(t) + B_w \tau_m(t) \quad (4)$$

$$v_s(t) = C_w \dot{\theta}_s(t) - D_w \tau_c(t)$$

and

$$v_m(t) = C_w \dot{\theta}_m(t) - D_w \tau_m(t) \quad (5)$$

$$u_s(t) = A_w \dot{\theta}_s(t) + B_w \tau_c(t)$$

where  $A_w$ ,  $B_w$ ,  $C_w$ , and  $D_w$  are  $n \times n$  wave variable scaling matrices and  $n$  is the number of degrees of freedom of the teleoperation system. The subscript 'w' denotes the fact that the scaling matrices correspond to wave variable coefficients. These matrices cannot be chosen arbitrarily; certain relationships must hold so that the proper power relationships hold, e.g., the power flow for the master side should be

$$\dot{\theta}_m^T \tau_m = \frac{1}{2} u_m^T u_m - \frac{1}{2} v_m^T v_m \quad (6)$$

and for the slave side

$$\dot{\theta}_s^T \tau_c = -\frac{1}{2} u_s^T u_s + \frac{1}{2} v_s^T v_s. \quad (7)$$

Furthermore, one must determine conditions for the scaling matrices to guarantee passivity [9], [22].

Substituting equations (4) and (5) into equation (6) or (7), expanding, and matching matrix coefficients yields the requirements

$$A_w^T A_w = C_w^T C_w \quad (8)$$

$$B_w^T B_w = D_w^T D_w$$

and also that

$$A_w^T B_w + C_w^T D_w = I. \quad (9)$$

### 2.4 Rules for Determining Scaling Matrices

To determine the complete set of scaling matrices, we first derive the whole family of matrices satisfying (8) and (9). In order to do this, we first use (8) to relate  $C_w$  and  $D_w$  to  $A_w$  and  $B_w$ , respectively, and then apply (9) to relate  $A_w$  and  $B_w$ . In previous work [1], we determined the necessary and sufficient conditions for  $A_w$ ,  $B_w$ ,  $C_w$ , and  $D_w$  that satisfy (8) and (9):

1.  $A_w$  is nonsingular.
2.  $B_w = \frac{1}{2}(I + S)A_w^{-T}$  where  $S$  is any  $n \times n$  skew-symmetric matrix.
3.  $C_w = QA_w$  where  $Q$  is any  $n \times n$  orthogonal matrix.
4.  $D_w = \frac{1}{2}Q(I - S)A_w^{-T}$ .

Note that these four conditions guarantee that all four matrices are nonsingular. These conditions can be checked simply by substituting them back into (8) and (9).

At this stage, it has only been shown that the conditions given in this section characterize the family of scaling matrices that result in wave variables (4) and (5) that satisfy the power flow equations (6) and (7). Characterizing the family of scaling matrices that result in passivity requires more work. Next, we derive the input-output relationship across the communication link then we use scattering theory to prove passivity.

## 2.5 The Input-Output Relationship

The input-output relationship across the communication link has the form [14]

$$\begin{bmatrix} T_m \\ s\Theta_{sd} \end{bmatrix} = G_w(s) \begin{bmatrix} s\Theta_m \\ -T_c \end{bmatrix}. \quad (10)$$

The transfer function  $G_w(s)$  determines the stability of the system and is based on (1) and the wave variable relationships (4) and (5), also, we now assume that  $T_R$  and  $T_L$  may be different. In terms of Laplace transforms, the multiple degree-of-freedom version of (1) is given by

$$\begin{bmatrix} U_s(s) \\ V_s(s) \end{bmatrix} = \begin{bmatrix} e^{-sT_R} I & 0 \\ 0 & e^{sT_L} I \end{bmatrix} \begin{bmatrix} U_m(s) \\ V_m(s) \end{bmatrix}. \quad (11)$$

Substituting in the wave variable relationships (4) and (5) and rearranging [1] one obtains

$$G_w(s) = \begin{bmatrix} 2A_w^T e^{-sT_d} & 0 \\ 0 & A_w^{-1} \end{bmatrix} \hat{G}_w(s) \begin{bmatrix} 2A_w e^{sT_d} & 0 \\ 0 & A_w^{-T} \end{bmatrix} \quad (12)$$

where

$$\hat{G}_w(s) = \frac{1}{2} \begin{bmatrix} [I - S_w \tanh(sT_a)]^{-1} & 0 \\ 0 & [I - S_w \tanh(sT_a)]^{-1} \end{bmatrix} \times \begin{bmatrix} \tanh(sT_a)I & -\sec h(sT_a)I \\ \sec h(sT_a)I & \tanh(sT_a)(I + S_w^T S_w) \end{bmatrix} \quad (13)$$

and where  $T_a = (T_L + T_R)/2$  and  $T_d = (T_L - T_R)/2$ . Note that the orthogonal matrix  $Q$  does not appear in the expression for  $G_w(s)$  or  $\hat{G}_w(s)$  and hence has no effect on the input-output characteristics.

## 2.6 The Complete Family of Scaling Matrices that Result in Passivity

Now that the input-output relationship  $G_w(s)$  across the communication link has been determined, it is possible to study the stability of the system. Like previous work on the wave variable method, this will be done using passivity theory and the scattering operator. Applying these techniques, it will be shown that the family of scaling matrices derived earlier not only satisfy the power flow equations (6) and (7), but also result in stability for any constant time delays  $T_L$  and  $T_R$ .

The scattering matrix

$$S(j\omega) = [G_w(j\omega) - I][G_w(j\omega) + I]^{-1} \quad (14)$$

can be used to test the passivity of the system. It was shown in [2] that a system with transfer function  $G_w(s)$  is passive if and only if the norm

$$\|S\| = \sup_{\omega} \sqrt{\lambda_{\max}(S^*(j\omega)S(j\omega))} \quad (15)$$

of its scattering matrix is less than or equal to one, where  $\lambda_{\max}(S^*(j\omega)S(j\omega))$  denotes the largest eigenvalue of the positive definite (semi-definite) Hermitian matrix  $S^*(j\omega)S(j\omega)$ .

With some work [1], it can be shown that the family of scaling matrices derived previously result in passivity. Since choosing a set of scaling matrices requires the selection of an  $n \times n$  nonsingular matrix  $A_w$ , an  $n \times n$  orthogonal matrix  $Q$ , and an  $n \times n$  skew-symmetric matrix  $S_w$ , there are a total of  $2n^2 - n$  degrees of freedom in choosing the scaling matrices.

It is natural to ask how this new extension of the scaling matrices affects the wave variables. To see this, we first consider the effect of  $Q$ . The  $v$  wave variables are given in equations (4) and (5). Substituting in the expressions for  $C_w$  and  $D_w$  yields

$$\begin{aligned} v_m(t) &= QA_w \dot{\theta}_m(t) - \frac{1}{2} QA_w^{-T} \tau_m(t), \\ &= Q[A_w \dot{\theta}_m(t) - \frac{1}{2} A_w^{-T} \tau_m(t)] \end{aligned} \quad (16)$$

which clearly demonstrates that  $Q$  merely applies an orthogonal transformation to the  $v$ -variable, i.e., it will merely rotate and/or reflect the  $v$ -variable. The same holds for  $v_s(t)$ . This will clearly have no effect on the power flow equations (6) and (7).

While the orthogonal matrix  $Q$  has no external effect on the behavior of the system, the matrices  $A_w$  and  $S_w$  have a significant effect. As in the scalar case, the matrix  $A_w$  affects the damping of the system. The matrix  $S_w$ , which is not present in the scalar case, has several effects on the system output. In the next section we will describe an experiment designed to test how these matrices affect the user's ability to control the system.

## 3. OBJECTIVES OF EXPERIMENT

The intention of this study is to show the effects on human tracking performance when changing the scaling matrices in a teleoperation system with time delay and the wave variable method implemented. The effects the different scaling matrices have on accuracy as well as speed of completion are of interest. This tracking task may represent a remote flying situation. The pilot may have to track an object with a remote controlled aircraft in the presence of a time delay in the communication channel.

Through the simulations and experiments discussed previously, we have shown that while the orthogonal matrix  $Q$  has no external effect on the behavior of the system, the matrices  $A_w$  and  $S_w$  affect the system significantly. We have seen that as the norm of  $A_w$  increases, the system damping tends to decrease. By this we mean that the slave system tends to react faster but may also include some oscillation and/or overshoot. Simulations have also shown that as the matrix  $S_w$  increases in norm, the master and slave tended to take longer to come to a steady state. Also there is an initial difference that tends to increase as  $S_w$  does. Adding  $S_w$  gives the ability to affect individual dimensions and properly selecting a good combination of  $A_w$  and  $S_w$  matrices can result in a better performance in terms of overshoot and settling time than varying  $A_w$  alone.

The main objective of this experiment is to determine the best set of scaling matrices for the specific tracking task, as determined by

the human subjects. We also want to find out how the users feel about the system with the different sets of scaling matrices. This will be done by changing both the  $A_w$  and the  $S_w$  matrices between trials, collecting the data from the different trials, and asking the subjects to answer several questions about their experiences with the system. Finally, we will determine if the matrix sets chosen by the subjects as easiest to use yield good results.

## 4. METHOD

### 4.1 Equipment

For these experiments the PHANToM Omni haptic device was used as both the master and slave; one is shown in Figure 2. The Omni is a three degree-of-freedom robot with three revolute joints. The Omni allows for position and rotation information to be collected from the device as well as force data to be transmitted back to the device. As seen in Figure 2, the Omni has a wand attached to the end of the device that allows the user to position the arm of the robot. In the experiments the subjects used this wand as a pointer and the means to track a path.



Figure 2. Three degree-of-freedom PHANToM Omni haptic device.

Both the master and slave robots were hardwired to a central computer. Matlab, and more specifically SIMULINK was used in conjunction with the haptic devices. SIMULINK read in the velocity information from both the master and the slave PHANToM and transmitted the force information to the master and slave. SIMULINK also provided us the ability to add a time delay into the communication channel as well as implement the wave variable method as a means to stabilize the overall system. With the wave variable method in place we were able to change both the time delay amount and the parameters of the wave transformations, between trials, in order to determine the best set of scaling matrices for the experimental tasks.

### 4.2 Subjects

For this experiment a total of seven people participated. Several subjects were chosen from both the undergraduate as well as graduate students at the University College of Engineering. The rest of the subjects consisted of current and former University

Table 1. Demographics of subjects used in the study.

Gender – Male or Female	Age in years	Title
Female	23	Graduate student
Female	24	Former student
Female	25	Former student
Male	21	Undergraduate student
Male	25	Graduate student
Male	26	Graduate student
Male	26	Former Student

students from other disciplines. Table 1 shows some of the demographics of the subjects involved in the study. The subjects were given no compensation for completing the experiment.

## 4.3 Experimental Design

### 4.3.1 Performance Task

The experiment consisted of several tracking tasks. The subjects used the wand of the master PHANToM to trace three different shapes; they are shown in Figure 3. For each trial the subject was asked to trace all three shapes in order pausing slightly between each shape. Before the experiment began, the wand of the master PHANToM was placed on the ‘start’ line of the first shape. The subject was asked to begin each shape at the designated ‘start’ line. As the user traces the shapes the slave PHANToM should follow a similar path. In order to determine the best set of scaling matrices for the tasks, we changed the  $A_w$  matrices as well as the  $S_w$  matrices between each trial. We included two different amounts of time delay in the communication channel and determined the best set of matrices for each amount of delay. For the experiment there were four different  $A_w$  matrices and three different  $S_w$  matrices for a total of twelve different scaling matrix sets. For each amount of delay the matrix sets were presented in random order, and the users were asked to trace each of the three shapes.

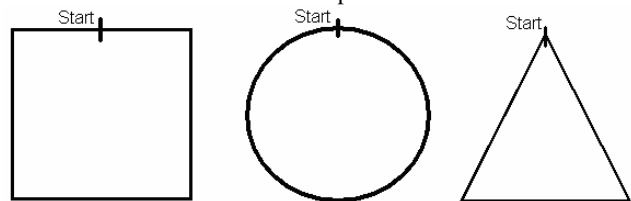


Figure 3. Three shapes traced by the subjects in the experiment

### 4.3.2 Training

Before the subjects were given the actual experimental tasks they were put through a short training regiment. First, the subjects were asked to run the ‘Dice’ demo. This program uses the PHANToM as the master and has a virtual slave. The demo allowed the subject to move a die in a three dimensional space, letting them become accustomed to using the wand of the PHANToM Omni while feeling the force feedback in the system. Once the subject felt that he or she was comfortable with the force feedback, the teleoperation system using two PHANToM Omnis was employed. Now that the hardware system was being used, the subject was asked to trace the three shapes using the wand of the master PHANToM. This first trial consisted of the teleoperation system with no time delay and the wave variable method not implemented in order to set a baseline for the experiment. After running the baseline condition, a time delay was added to the system, the wave variable method was implemented, and a random scaling matrix set was used.

### 4.3.3 Questionnaire

Once the subjects completed tracing the shapes for all of the different scaling matrix sets they were given a questionnaire. The same questions were given for each different amount of time delay. The questions asked were intended to determine whether the subjects felt they were given enough training to complete the tasks, the quality and difficulty of the experimental tasks, and most

importantly which set of scaling matrices they felt was best for the tasks. It is also important to note that the subjects were given the questions before the experiment began, and they were asked to keep them in mind while completing the different trials. However, we did not offer them any help on how to keep track of their feelings on the different sets of matrices. Table 2 shows the actual questions that were given to the subjects.

**Table 2. Questions given to subjects.**

Did you feel that you were comfortable enough with the PHANToM Omni to be able to complete the experiment successfully?
Did you feel that this experiment allowed you to differentiate between the conditions?
How would you rate the difficulty of completing the tasks in the experiment (1-10, 1 - easy, 10 - difficult)?
Please rank in order, from worst to best, the sets of conditions with respect to accuracy and then again with respect to speed.
Which set of condition, in general, did you feel was the easiest to use and why?

#### 4.3.4 Data Analysis

While the subjects completed the tracing task the three dimensional force, position, and velocity vectors were recorded for both the master and the slave PHANToM Omnis. This was done for each amount of time delay and for every scaling matrix set. The data from the master and the slave will be compared for each trial to determine the difference between the two devices for each of the three vectors. For each of the trials we will consider both the maximum difference as well as the total difference between master and slave. The total difference will be computed by summing the area under the curve of the absolute value of the difference vectors. Because we are concerned about both speed and accuracy, we also take into account the time of completion for each trial. For each amount of time delay we used the raw data to determine the best set of scaling matrices both for accuracy and for speed. This objective data will only provide us with part of the knowledge of the system's performance from trial to trial. As mentioned earlier we will also consider the subjective data collected from the questionnaire given to the subjects. The answers to the questionnaire provided us with the scaling matrix sets that the subjects felt were best for speed and accuracy for each amount of time delay. We used a weighted combination of the subjects' answers with the raw data in order to determine the overall best scaling matrix sets for the bilateral teleoperation tracking task.

## 5. Experimental Results

### 5.1 Raw Data

In the following section, we will discuss the objective results of the experimental trials. We will begin by showing and explaining the raw data collected during the different subjects' trials. Table 3 shows the results of all the trials with a 400msec total time delay, and Table 4 shows the results of the system with 1sec total time delay. All results shown are averages of all test subjects experimental runs. The individual results were determined and then averaged.

For most applications the position of the slave with respect to the master is the most important factor. Because of this, the tables display the maximum position difference and the total position difference as well as the time of completion for each set of scaling

matrices. Each scaling matrix set is defined by a combination of an  $A_w$  matrix as well as an  $S_w$  matrix; all matrices used in the experiment can be found in Appendix A. Also, as a reference the baseline trial with no time delay and the wave variable method not implemented had the following average results:

maximum position difference=0.0028m  
total position difference=0.0577m  
time of completion=13.73sec

**Table 3. Average of the raw data from the human trials with 400msec total time delay.**

Matrix Set	Max. Pos. Diff.	Total Pos. Diff	Time of Comp.
$A_{w1}, S_{w1}$	0.0602m	0.8949m	18.03sec
$A_{w2}, S_{w1}$	0.0153m	0.2134m	16.89sec
$A_{w3}, S_{w1}$	0.0103m	0.1242m	17.61sec
$A_{w4}, S_{w1}$	0.0069m	0.1004m	18.58sec
$A_{w1}, S_{w2}$	0.1067m	1.4263m	19.42sec
$A_{w2}, S_{w2}$	0.0374m	0.5166m	18.19sec
$A_{w3}, S_{w2}$	0.0181m	0.2962m	18.97sec
$A_{w4}, S_{w2}$	0.0173m	0.2595m	19.83sec
$A_{w1}, S_{w3}$	0.0646m	0.9977m	15.80sec
$A_{w2}, S_{w3}$	0.0203m	0.2810m	14.46sec
$A_{w3}, S_{w3}$	0.0126m	0.2022m	15.25sec
$A_{w4}, S_{w3}$	0.0106m	0.1413m	16.34sec

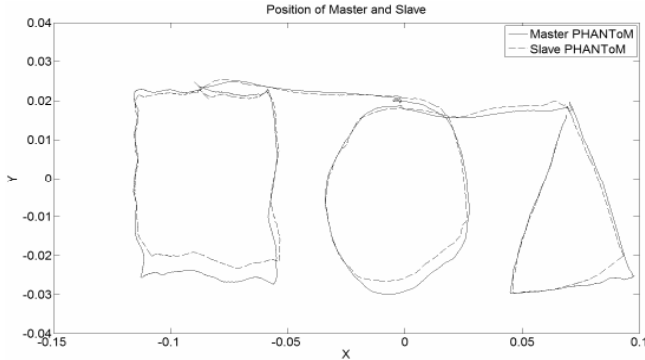
**Table 4. Average of the raw data from the human trials with 1sec total time delay.**

Matrix Set	Max. Pos. Diff.	Total Pos. Diff	Time of Comp.
$A_{w1}, S_{w1}$	0.1210m	1.6923m	18.46sec
$A_{w2}, S_{w1}$	0.0278m	0.4123m	16.23sec
$A_{w3}, S_{w1}$	0.0105m	0.1289m	17.54sec
$A_{w4}, S_{w1}$	0.0174m	0.1845m	16.91sec
$A_{w1}, S_{w2}$	0.1817m	1.9967m	21.97sec
$A_{w2}, S_{w2}$	0.0688m	0.9968m	19.89sec
$A_{w3}, S_{w2}$	0.0218m	0.3539m	21.03sec
$A_{w4}, S_{w2}$	0.0347m	0.5785m	20.47sec
$A_{w1}, S_{w3}$	0.1313m	1.8517m	18.92sec
$A_{w2}, S_{w3}$	0.0363m	0.5351m	16.86sec
$A_{w3}, S_{w3}$	0.0145m	0.2045m	18.01sec
$A_{w4}, S_{w3}$	0.0210m	0.3169m	17.44sec

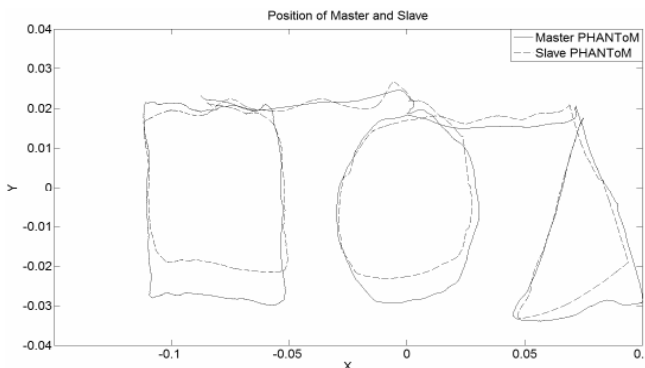
#### 5.1.1 Accuracy

As mentioned earlier, in order to determine the best set of scaling matrices for accuracy we examined both the maximum position difference as well as the total position difference for each trial. As can be seen in Table 3, the set of scaling matrices with both the lowest maximum and total position difference is  $A_{w4}, S_{w1}$ . From Table 4 it can be seen that the system with the lowest differences used  $A_{w3}, S_{w1}$ . We attribute this to the damping of the system. For the system with less time delay, the scaling matrix set that provides the least damping proved to be the best, meaning the other systems were overdamped. However, once more time delay was added into the system, more damping was required to yield the best results. The best matrix set for the system with 400msec delay caused the system with 1sec to be underdamped.

Table 3 and Table 4 also show that even though the four sets of matrices using  $S_{w1}$  overall were better, there were combinations with  $S_{w3}$  that individually yielded better results than combinations using  $S_{w1}$ . Figure 4 shows the paths taken from one subject's trial



**Figure 4. Paths taken when three shapes were traced by one subject during the experiment using the best, in terms of accuracy, set of scaling matrices,  $A_{w4}, S_{w1}$ , for a  $2T=400msec$  time delay.**



**Figure 5. Paths taken when three shapes were traced by one subject during the experiment using the best, in terms of accuracy, set of scaling matrices,  $A_{w4}, S_{w1}$ , for a  $2T=1sec$  time delay.**

with a total time delay of  $400msec$  and the scaling matrix set  $A_{w4}, S_{w1}$  implemented. The figure shows that the slave PHANToM had very similar path to that of the master, only slightly delayed in time. Figure 5 shows the paths taken from one subject's trial with a total time delay of  $1sec$  and the scaling matrix set  $A_{w3}, S_{w1}$  implemented. Looking at the paths taken by the master and slave, we see that there was a small amount of oscillation at times caused by the larger amount of time delay.

### 5.1.2 Speed

When trying to determine the best set of scaling matrices for speed, we used the performance measure time of completion. Examining Table 3 and Table 4 it can be seen that  $A_{w2}, S_{w3}$  and  $A_{w2}, S_{w1}$  are the best for the respective amounts of time delay. Because the best matrix sets for speed are different from those for accuracy, we can see that there is a tradeoff between the two. As the accuracy got worse, the time of completion tended to improve. However, if the accuracy became too poor, the time of completion went up due to the fact that the system was hard to control properly. We would also like to note that there was a greater spread between different subjects in the time of completion data compared to that of the position difference results. However, the individual subject's results for the fastest completion times were the same as the average results shown.

## 5.2 Objective and Subjective Data Comparison

In order to complete the analysis of the experiment we will discuss the subjects' answers from the questionnaire. The first three questions were used as a means to ensure that the experiment was adequately constructed. From the answers given to question one, every subject felt comfortable with the PHANToM Omni before the experimental trials began. This confirms that the training process was successful. Also, from question two we found that the subjects all felt that they could, in most cases, differentiate between the matrix sets. This shows that the matrix sets resulted in significantly different outcomes. The average answer for question three was 4 out of 10 for the system with  $400msec$  total time delay, and as expected, it rose to 6 out of 10 for the system with  $1sec$  delay.

The next two questions were used to determine which sets of scaling matrices the subjects thought were best for both speed and accuracy. When comparing the answers to question four with the results in Table 3 and Table 4, we found the same general trends. In terms of accuracy, 6 out of 7 subjects ranked  $A_{w4}, S_{w1}$  as the best condition with  $400msec$  time delay; also 6 out of 7 subjects chose  $A_{w3}, S_{w1}$  for the system with  $1sec$  time delay. When looking at time of completion, all 7 subjects ranked  $A_{w2}, S_{w3}$  as the best for speed with  $400msec$  time delay, and 5 out of 7 subjects chose  $A_{w2}, S_{w1}$  for the system with  $1sec$  time delay. In general the rankings given by the subjects matched closely with their actual results as measured by the performance criteria maximum position difference, total position difference, and time of completion.

We determined the best overall matrix sets based on the combination of the raw data and the answers to question five. For the system with  $400msec$  time delay, 4 out of 7 subjects chose  $A_{w4}, S_{w3}$  as the overall easiest to use. Table 3 shows that this scaling matrix set is third best in both accuracy categories and fourth best in speed. For the system with  $1sec$  total time delay, 5 out of 7 subjects thought that  $A_{w4}, S_{w1}$  was the easiest to use. Using Table 4, this scaling matrix set was second best for total position difference and third best for both maximum position difference and time of completion. Since the subjective results closely resembled the objective results, we considered the experiment to be successful in determining the best set of scaling matrices for the PHANToM Omni teleoperation system with both amounts of time delay.

## 6. CONCLUSION

In this article, we have utilized a generalized version of the wave variable method which includes the complete family of scaling matrices. We have used the extended family of scaling matrices in an experiment with human subjects and a PHANToM Omni haptic teleoperation system with time delay. The subjects in the experiment were asked to trace three shapes, between each trial we changed the scaling matrix set that was used. Also, the experiment was completed for two different amounts of time delay. The analysis compared the raw data with the users' opinions in order to determine the best set of scaling matrices for the tracing task. Once all subjects completed the experiment, the analysis clearly determined the best set of scaling matrices for each different performance measure. Also, we determined that the most user friendly scaling matrix sets also provided good overall system results.

The PHANToM Omni haptic teleoperation provides a testbed for numerous future experiments. The experimental task can be changed to a three dimensional one, an unknown or varying time delay can be added, and/or the method of dealing with the time delay can be changed. Any combination of these changes will provide further insight into and knowledge of teleoperation systems in the presents of time delay.

## 7. APPENDIX – LIST OF SCALING MATRIX SETS

$$A_{w1} = \begin{bmatrix} 2 & -0.5 & -0.5 \\ -0.5 & 2 & -0.5 \\ -0.5 & -0.5 & 2 \end{bmatrix} \quad (A1)$$

$$A_{w2} = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & 4 \end{bmatrix} \quad (A2)$$

$$A_{w3} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad (A3)$$

$$A_{w4} = \begin{bmatrix} 8 & 2 & 1 \\ 2 & 10 & 2 \\ 1 & 2 & 12 \end{bmatrix} \quad (A4)$$

$$S_{w1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (A5)$$

$$S_{w2} = \begin{bmatrix} 0 & -0.5 & -1 \\ 0.5 & 0 & -0.5 \\ 1 & 0.5 & 0 \end{bmatrix} \quad (A6)$$

$$S_{w3} = \begin{bmatrix} 0 & 0.5 & -0.25 \\ -0.5 & 0 & 0.5 \\ 0.25 & -0.5 & 0 \end{bmatrix} \quad (A7)$$

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